CONSTRICTION OF JETS OF NEWTONIAN LIQUID ISSUING FROM CAPILLARY NOZZLES UNDER CONDITIONS OF LARGE SHEAR STRESS VALUES

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A new approximate method of solution of a current engineering problem is described.

In recent years the number of publications on discharge of liquid jets from capillary nozzles has increased. The interest in this topic is due mainly to the fact that such jets are used as an investigative tool in some regions of chemistry, physics, and chemical technology [1, 2].

In investigations of the hydrodynamics of a free jet of liquid the following basic features are usually dealt with: the flow stability, the distribution of velocities and stresses under the action of external extensive forces, and the relaxation of the velocity profile at the exit from the capillary. This paper examines a third group of problems, namely the phenomenon of constriction of a jet of liquid upon discharge into a medium with very low viscosity. In this case the initial velocity profile is brought to equilibrium and becomes homogeneous as a result of the action of viscous forces within the jet. The relaxation phenomenon proceeds very rapidly, and a free jet is formed at a section whose length is only a few capillary diameters [2].

Until recently it was assumed that the only characteristic feature of such a flow is constriction of the jet due to flattening of the velocity distribution. The constriction coefficient has been determined theoretically by Harmon [4], who, on the basis of a simple balance of forces acting on the jet, established the following relation for a horizontal jet:

$$\varkappa = \frac{d_f}{d_0} = \frac{\sqrt{3}}{2} = 0.867, \qquad (1)$$

valid for a parabolic initial velocity distribution. Middleman and Gavis [1], in 1951 established experimentally the hitherto unexpected effects of growth of diameter of the jet for small Reynolds numbers. Strictly speaking, the dilation of a jet of viscoelastic liquid was known earlier, but this phenomenon was attributed to partial elasticity, and it was not assumed that it could also appear in the case of Newtonian liquids. Therefore Harmon's analysis required an addition, which Middleman and Gavis [1] contributed. They took account of the previously ignored action of surface forces and as a result of reasoning based partially on dimensional analysis, they derived the following equation:

$$\frac{1}{\varkappa^2} = \frac{4}{3} - \frac{8N}{\mathrm{Re}} - \frac{2\varkappa}{\mathrm{We}^2}, \qquad (2)$$

where N is a definite integral. Equation (2) shows that the residual diameter of a free jet is a function of the Reynolds and Weber numbers. It is easy to see that for considerable jet velocities, when the Re and We numbers are large, Eqs. (2) and (1) coincide.

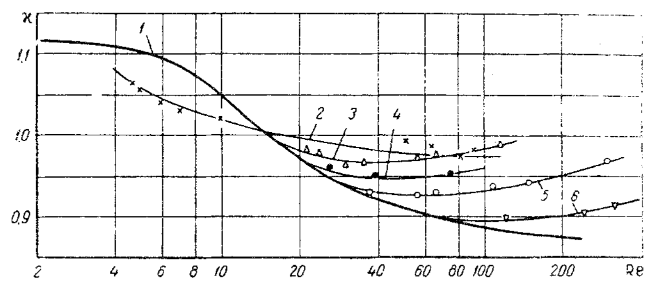

Fig. 1. The influence of pressure drop in a nozzle on the value of the coefficient ϰ: 1—according to Middleman and Gavis [1]; 2—for castor oil, d_0 = = 120 cm, L = 4.9 cm, p = 33–120 atm. abs.; 3—the same, d_0 = 216 cm, L = 4.2 cm, p = 21–70 atm. abs.; 4—for glycerine, diluted with water (7%), d_0 = 120, L = 4.9, p = 41–71; 5—the same, d_0 = 0.818, L = 4.9, p = 21–93; 6—the same, d_0 = = 0.269, L = 4.3, p = 18.5–53.

In their tests the authors of [1] determined the dependence of the coefficient of variation of diameter ϰ on Re by finding the characteristic regions of expansion and contraction of a jet (curve 1 in Fig. 1). On the basis of the results obtained it may be assumed that Harmon's [4] simple analysis is valid for the region Re > 150.

DISCHARGE OF A LIQUID FROM CAPILLARY NOZZLES AT HIGH PRESSURE

The publications presently available refer to jets of liquids discharging from nozzles under conditions of developed laminar velocity distribution. If the passage of the liquid through the capillaries requires large pressure drops, i.e., is accompanied by considerable shear stresses, this assumption becomes satisfied. In this case the thermal effect of energy dissipation and of expansion of the liquid have a definite influence on the formation of the initial velocity profile of the free jet leaving the nozzle. Measurements of the coefficient ϰ during discharge of a liquid from a nozzle at high pressure [4] indeed show an appreciable influence of pressure drop on the expansion or contraction of the jet (Fig. 1, curve 1).

Determination of the residual diameter of the jet in such conditions requires account to be taken of the conditions of flow through the nozzle.

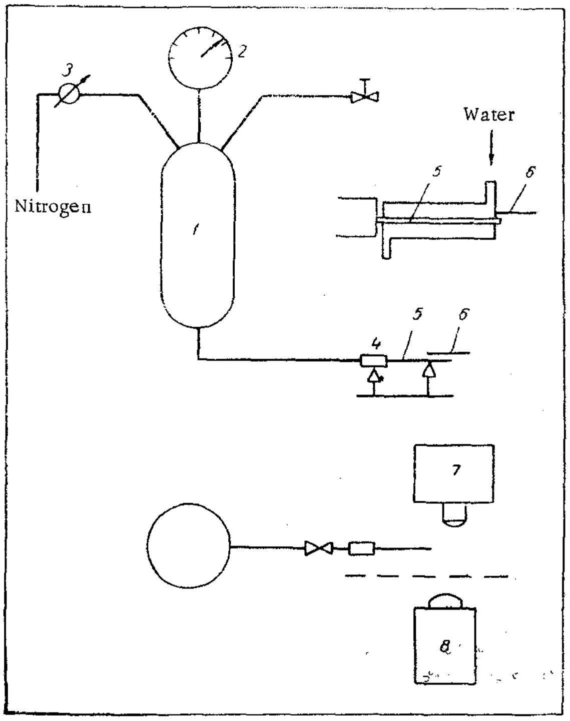

Fig. 2. Schematic of the equipment.

Our attempt to solve this problem of contraction of jets leaving a nozzle at large velocity (Re > 150) is presented below.

EXPERIMENTAL DETAILS

Experimental measurements of the dependence of the constriction coefficient on the pressure drop of the liquid in the nozzle have been made, using a capillary whose temperature is maintained at that of the oncoming liquid. The equipment devised by Middleman and Gavis [1] (Fig. 2) consists of a delivery reservoir to supply the liquid 1, a manometer 2 with range 0–100 atm. abs. and 0.5 atm. abs. scale divisions, a reducing valve 3, a bottle of compressed nitrogen, and a fitting 4, to which is attached the capillary nozzle 5. The temperature of the capillary nozzle is held constant by means of a water jacket. A camera 7 and a flash lamp 8 are used to determine the jet diameter. The jet is photographed along with a standard calibrated wire 6. Since a lamp with a flash duration of 10^{-3} sec was used, the equipment was located in the basement of the building to eliminate the possible action of constant vibrations.

The picture size obtained with the small camera was 1–2 mm. The diameters of the jet and of the standard wire were measured with an optical comparator to an accuracy of 0.005 mm, i.e., 0.5%. The working liquid was a 73-per cent aqueous solution of glycerin, extruded through a capillary nozzle. The capillary was a steel hypodermic needle (calibrated) of diameter $d_0 = 0.0625$ cm, length $L = 10.0$ cm. Its outlet was specially ground.

The physical properties of the liquid in the test conditions were: $T_0 = 19.40$; $\mu(P) = 0.309$; $\lambda = 3.42 \cdot 10^4$; $\gamma = 1.187$, $c_p = 2.93 \cdot 10^7$.

The remaining test parameters were varied in the range: Pressure $1.47–7.35 \cdot 10^7$ dyne/cm^2 (15–75 atm. abs.); Reynolds number (referred to the diameter d_0) 140–717.

The test data are presented in Fig. 3. A theoretical calculation of the coefficient $\varkappa$ was made by a method described below. On the basis of the analysis made at the beginning of the paper, it may be assumed, that for jets discharging from capillaries with Re > 150, inertia forces, not surface forces, play a decisive role. This is what justifies the simple force balance in Harmon's formulation [4].

By comparing the momentum at the initial section of the jet, i.e., at the nozzle lip, and at a section corresponding to the established diameter, we may write

$$M_0 = M_f, \tag{3}$$

$$2\pi R^2 \gamma \int_0^1 V^2 \rho \, d\rho = \pi R_f^2 V_f^2 \gamma. \tag{4}$$

The subscripts f refer to a section in the constant jet diameter region. The velocity V_f may be expressed by means of a velocity averaged over the capillary diameter.

$$V_f = \bar{V} \left(\frac{R}{R_f} \right)^2 = 2 \left(\frac{R}{R_f} \right)^2 \int_0^1 V \rho \, d\rho. \tag{5}$$

As a result, on the basis of (3), we obtain

$$\varkappa^2 = \left[2 \int_0^1 V \rho \, d\rho \right]^2 \Big/ 2 \int_0^1 V^2 \rho \, d\rho. \tag{6}$$

Equation (6) may be solved for a known liquid velocity distribution at the initial section of the jet. In the case of a parabolic velocity profile it simplifies to the form of (1).

The question of non-isothermal capillary flows has not as yet received sufficient attention in the literature. Therefore, use was made of an approximate method based on an independent solution of the equations of motion and energy [5].

Assuming that the liquid temperature distribution in the capillary is known, the physical parameters of the liquid in the equations of motion

$$\frac{dp}{dx} = \frac{1}{r} \frac{d}{dr} \left(\mu r \frac{dV}{dr} \right) \tag{7}$$

may be represented as a function of temperature. The equation does not take account of gravitational forces, and therefore the influence of temperature on the liquid density is not evaluated. Such a simplification in the case of capillary flows is justified, since the phenomenon of natural convection in a capillary plays no part in practice.

The dependence of the liquid viscosity on temperature may be represented by the following empirical formula:

$$\frac{1}{\mu} = \frac{1}{\mu'} (1 + \beta_1 t + \beta_2 t^2). \tag{8}$$

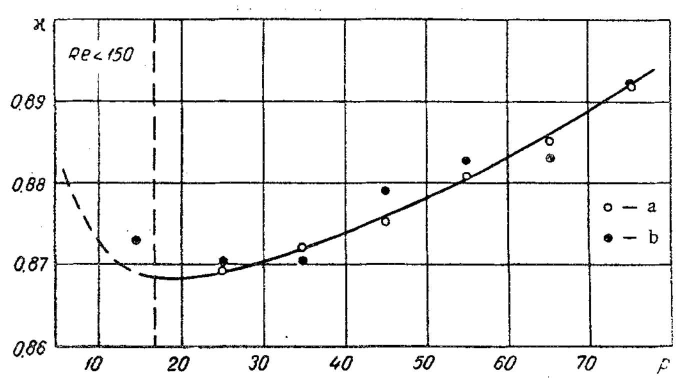

Fig. 3. Comparison of the contraction coefficient calculated (a) and determined experimentally (b).

in which t is the increase of local temperature of the liquid the excess of the local temperature of the liquid above a characteristic temperature T', determining the value of μ'.

In the case of the frequently examined capillary flows [5, 7] in a thermally-insulated or isothermal capillary (temperature controlled at the liquid entrance temperature) the reference temperature T' is equal to T_0.

Substituting (8) into (7), and replacing the pressure gradient by the value of the total pressure drop in the nozzle

$$\frac{dp}{dx} = \left(-\frac{P}{x}\right),$$

following repeated integration, we may write

$$\frac{dV}{d\rho} = \frac{R^2}{2\mu'}\left(-\frac{P}{x}\right)(1 + \beta_1 t + \beta_2 t^2)\rho, \quad (9)$$

and also

$$V = \left(\frac{P}{x}\right)\frac{R^2}{4\mu'}\left[(1-\rho^2) + 2\beta_1 \int_\rho^1 \rho t\, d\rho + 2\beta_2 \int_\rho^1 \rho t^2 d\rho\right]. \quad (10)$$

Integrating the numerator of (6) by parts, and then using (9) and (10), we obtain finally a relation for the contraction coefficient of the jet

$$\varkappa^2 = \frac{1}{8}\frac{\left[1 + 4\beta_1 \int_0^1 \rho^3 t\, d\rho + 4\beta_2 \int_0^1 \rho^3 t^2 d\rho\right]^2}{\int_0^1 \left[(1-\rho^2) + 2\beta_1 \int_\rho^1 \rho t\, d\rho + 2\beta_2 \int_\rho^1 \rho t^2 d\rho\right]\rho\, d\rho}. \quad (11)$$

In accordance with (11), the value of the contraction coefficient is a function of the liquid temperature distribution at the initial section of the free jet. In the case of isothermal flow (t = 0, T = T'), (11) is simplified to the form (1).

To determine the temperature profile at the nozzle lip use was made of an equation derived in reference [6], pertaining to laminar flow of Newtonian compressible fluids with allowance for friction heating. According to the computational method used there, we had to introduce into Eq. (8) a transformed temperature (according to Brinkman [7])

$$\tau = 16\mu\lambda T/R^4\left(\frac{P}{x}\right)^2 = 2\pi(\mathrm{Gr})^{-1}\left(\frac{c_p \gamma T}{P}\right); \quad T = k\tau. \quad (12)$$

Using the notation $\vartheta = \tau - \tau'$, relation (8) may be written in the form

$$\frac{1}{\mu} = \frac{1}{\mu'}(1 + \beta_1 k\vartheta + \beta_2 k^2 \vartheta^2). \quad (13)$$

In accordance with the premise that the physical parameters of the liquid are constant in deriving equations for the temperature distribution [6] the quantity k is constant, conventionally determined by the viscosity at the reference temperature (in the actual case of a capillary whose temperature is maintained at the initial liquid temperature, the reference temperature $\tau' = \tau_0$, and $\mu = \mu_0$).

Introducing (13) into Eqs. (9), (10), and (16), we obtain a new form of relation (11), containing the transformed temperature

$$\varkappa^2 = \frac{1}{8}\frac{\left[1 + 4\beta_1 k \int_0^1 \rho^3 \vartheta\, d\rho + 4\beta_2 k^2 \int_0^1 \rho^3 \vartheta^2 d\rho\right]^2}{\int_0^1 \left[(1-\rho^2) + 2\beta_1 k \int_\rho^1 \rho\vartheta\, d\rho + 2\beta_2 k^2 \int_\rho^1 \rho\vartheta^2 d\rho\right]\rho\, d\rho}. \quad (14)$$

Since the increase of local liquid temperature during the tests did not exceed a few degrees, the viscosity of the liquid was expressed as a function of temperature in the simplified form:

$$\frac{1}{\mu} = \frac{1}{\mu_0}(1 + \beta_1 k\vartheta). \quad (15)$$

The value of the coefficient β_1 in the conditions of measurement was 0.0595.

Equation (14) is simplified in the case when β_2 is omitted, and the contraction coefficient may be written in the form

$$\varkappa = 0.867 \times \frac{\left(1 + 4\beta_1 k \int_0^1 \rho^3 \vartheta\, d\rho\right)}{\sqrt{1 + 24\beta_1 k \int_0^1 (1-\rho^2) Q \rho\, d\rho + 24\beta_1^2 k^2 \int_0^1 Q^2 \rho\, d\rho}}, \quad (16)$$

where $Q = \int_\rho^1 \rho\vartheta\, d\rho$.

In (16) the second factor (fraction) is a correction which must be necessary taken into account with regard to equation (1) in calculating the contraction of jets discharging at high pressure.

The contraction coefficient was calculated by means of Eq. (16).

To determine the value of the integrals $\int_0^1 \rho^3 \vartheta \, d\rho$ and Q, use was made of the numerical tables presented in reference [6]. The expressions $\int_0^1 (1-\rho^2) Q \rho \, d\rho$ and $\int_0^1 Q^2 \rho \, d\rho$ appearing in the denominator were integrated numerically by Simpson's rule. The calculated values of the coefficient $\varkappa$ and the values found experimentally are shown in Fig. 3. A dotted line corresponding to the limiting pressure drop is shown, below which the jets discharge from nozzles with Re numbers less than 150. The experimental curve, coinciding as it does with the theoretical in the region Re > 150, confirms the validity of Eq. (16). Thus, the test results show the usefulness of formulas (11) and (14) in calculating contraction coefficients for jets of Newtonian liquids, discharging from capillary nozzles, with Reynolds numbers greater than 150. In the case of low values of shear stress, the equation is reduced to the simple form of Eq. (1), put forward earlier by Harmon.

NOTATION

c_p—specific heat, erg/°C; d_0—capillary diameter, cm; d_f—residual jet diameter, cm; Gr—Graetz number; k—constant of Eq. (12); L—length of the capillary nozzle, cm; M—specific momentum, cm/sec^2; P—pressure drop, dyne/cm^2, atm. abs.; R—capillary radius, cm; r—current radius, cm; R_f—residual jet radius; T—temperature, °C; T_0—initial liquid temperature, °C; t—temperature excess with regard to a reference (characteristic) temperature, t = T−T'; V—velocity along the axis, cm/sec; We—Weber number; x—coordinate along the channel axis, cm; β_1, β_2—coefficients in the viscosity formula (8); γ—density, gm/cm^3; τ—transform temperature in (12); τ_0—the initial transformed temperature of the liquid; φ—increment in the transformed temperature; $\varkappa$—contraction coefficient of the jet, $\varkappa = d_f/d_0$; λ—specific thermal conductivity, erg/cm·sec °C; μ—viscosity, gm/cm·sec; ρ—dimensionless radius, r/R.

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